Total Charge

Q: *If we know charge density $\rho_v(\bar{r})$, describing the charge distribution throughout a **volume** V, can we determine the **total charge** Q contained within this volume?*

A: You betcha! Simply **integrate** the charge density over the entire volume, and you get the **total charge** *Q* contained within the volume.

In other words:

$$Q = \iiint_V \rho_v(\bar{r})\, dv$$

Note this is a **volume integral** of the type we studied in Section 2-5. Therefore select the differential volume *dv* that is appropriate for the volume V.

Likewise, we can determine the total charge distributed across a **surface** S by integrating the surface charge density:

$$Q = \iint_S \rho_s(\bar{r})\, ds$$

Q: *Hey! This is **NOT** the surface integral we studied in Section 2-5.*

A: True! This is a **scalar** integral; sort of a two-dimensional version of the volume integral.

The differential surface element *ds* in this integral is simply the **magnitude** of the differential surface vectors we studied earlier:

$$ds = \left|\overline{ds}\right|$$

For example, if we integrate over the surface of a sphere, we would use the differential surface element:

$$ds = \left|\overline{ds}_r\right| = r^2 \sin\theta \; d\theta \; d\phi$$

Finally, we can determine the total charge on **contour** *C* by integrating the **line charge density** $\rho_\ell(\bar{r})$ across the entire contour:

$$Q = \int_C \rho_\ell(\bar{r})\, d\ell$$

The differential element $d\ell$ is likewise related to the differential displacement vector we studied earlier:

$$d\ell = \left|\overline{d\ell}\right|$$

For example, if the contour is a circle around the *z*-axis, then $d\ell$ is:

$$d\ell = \left|\overline{d\phi}\right| = \rho\, d\phi$$